DOUBLE DIFFUSIVE CONVECTION

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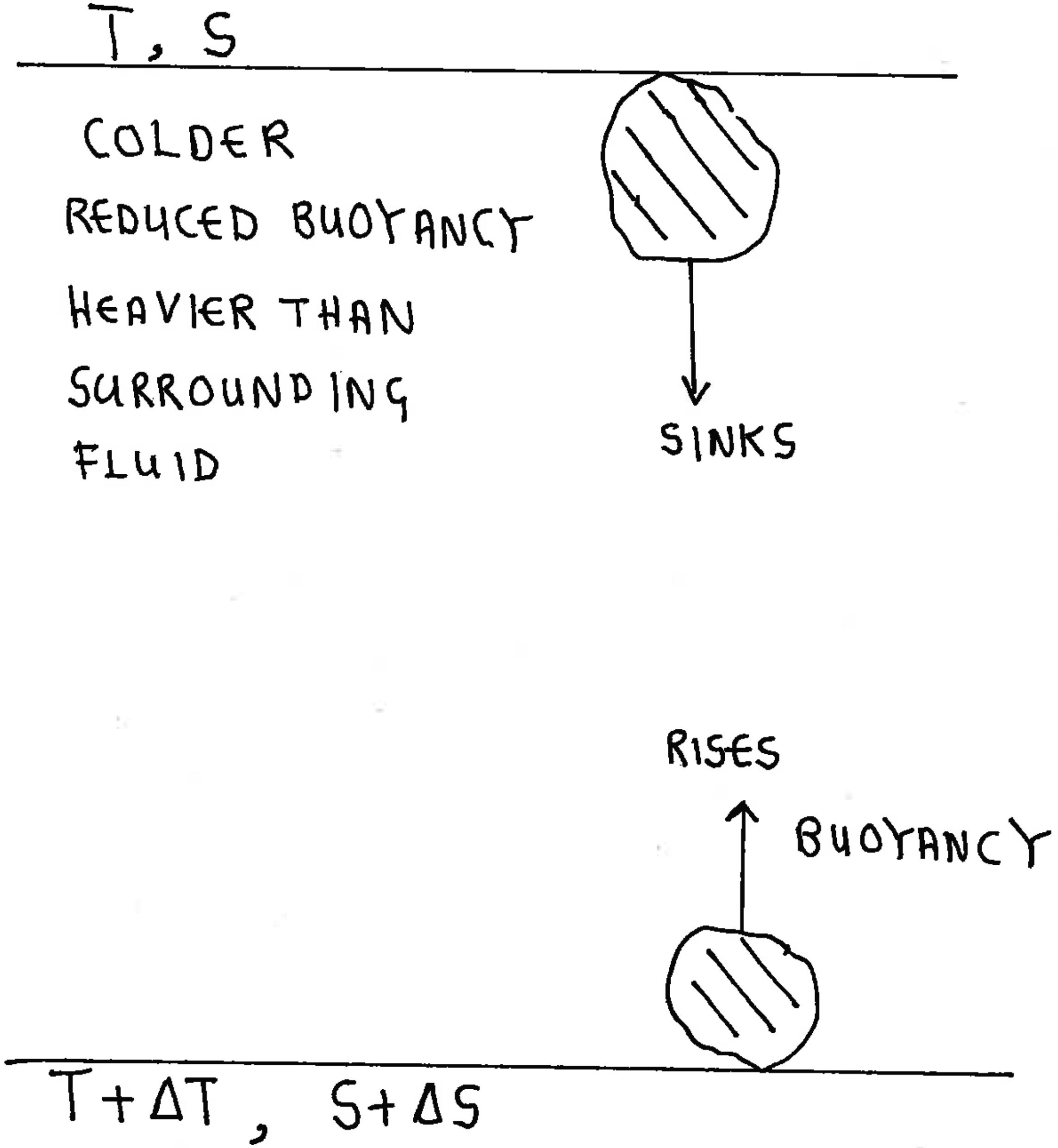
DOUBLE DIFFUSION OCCURS WHEN

- TWO COMPONENTS WITH DIFFERENT DIFFUSION COEFFICIENTS
- OPPOSING EFFECTS ON VERTICAL DENSITY GRADIENT

- OSCILLATORY MOTION



T = TEMPERATURE

S = SALT (DISSOLVED SUBSTANCES)

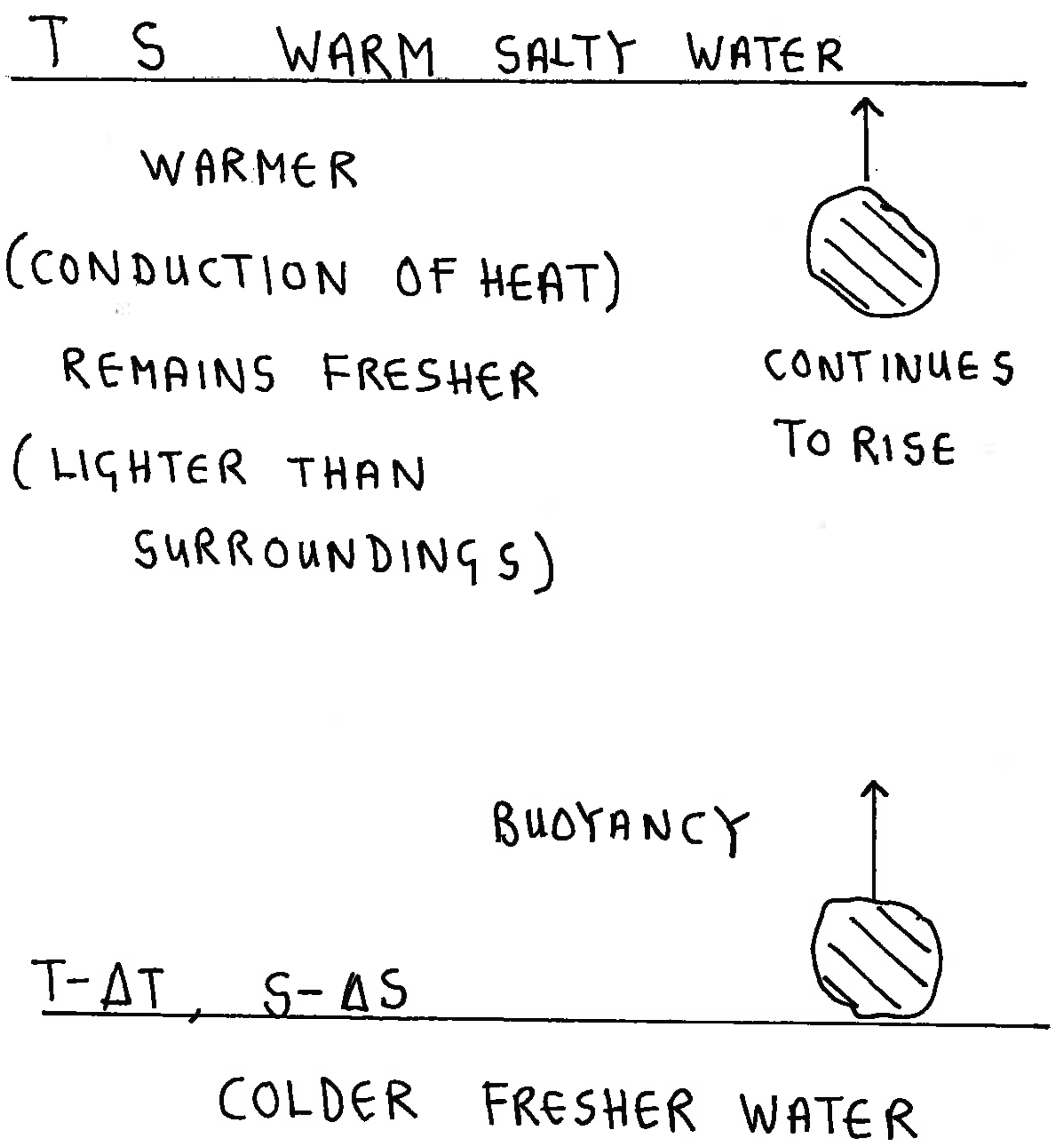
- FLUID ELEMENT LOSES HEAT BUT NOT SALT

  $K_S < K_T$

- BUOYANCY REVERSES IN EACH HALF CYCLE
- OSCILLATORY MOTION IS PRODUCED
- CAN OVERSHOOT POSITION EQUILIBRIUM

- A SERIES OF STEPS TEND TO FORM.
  WELL MIXED LAYERS.
  SEPARATED BY SHARPER DENSITY INTERFACES.

SALT FINGERS

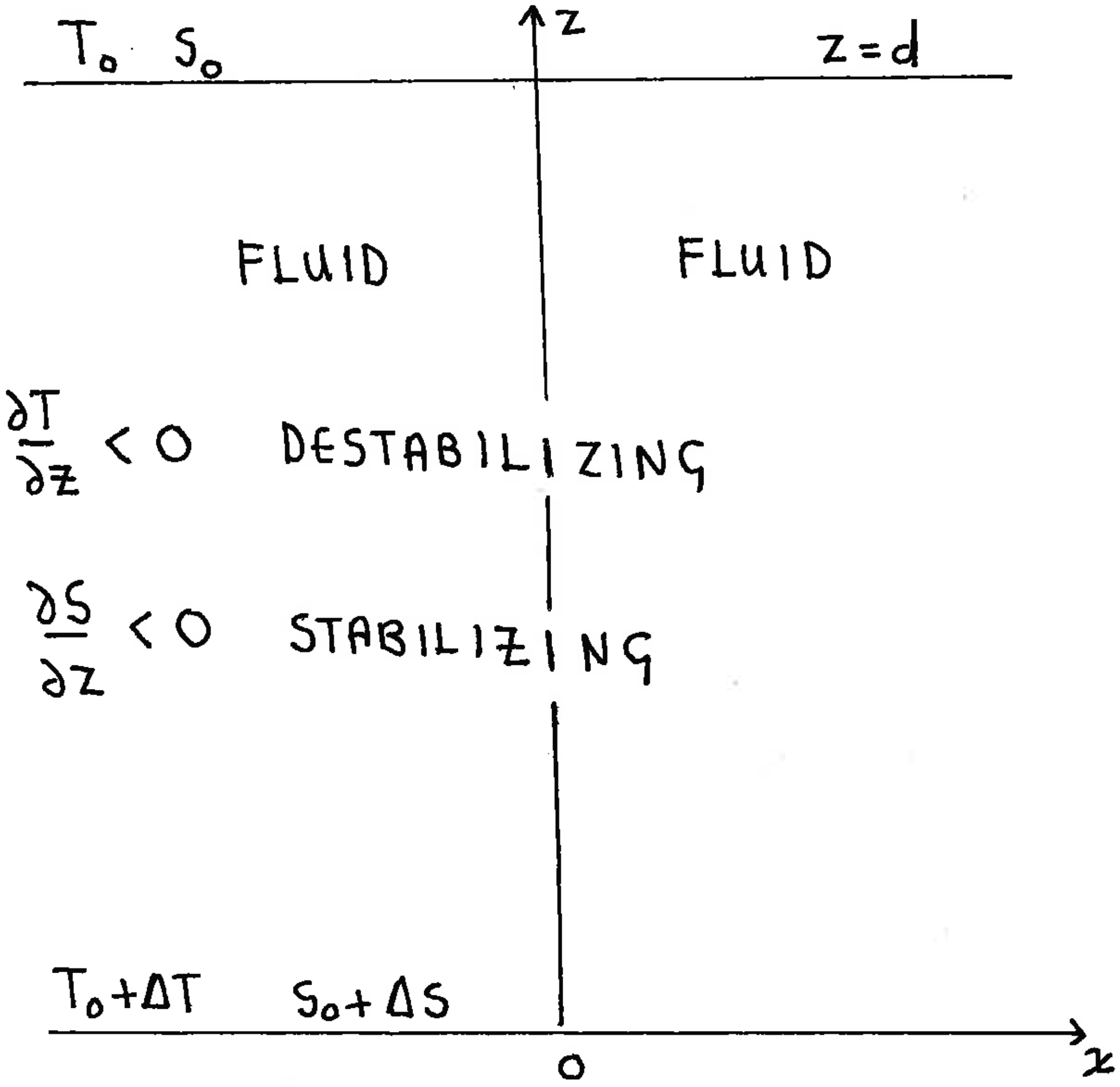


- DIFFUSION COEFFICIENT HEAT > DIFFUSION COEFFICIENT SALT
- LONG NARROW CELLS CALLED SALT FINGERS FORM
- SALT FOUNTAIN

SOME OF THESE PROPERTIES CAN BE DERIVED USING STABILITY ANALYSIS

- J S TURNER "BUOYANCY EFFECTS IN FLUIDS". CHAPTER 5.
- P G BAINES AND A E GILL "ON THERMOHALINE CONVECTION WITH LINEAR GRADIENTS" J FLUID MECHANICS (1969) 37, 289-306.
- H E HUPPERT AND D R MOORE "NONLINEAR DOUBLE-DIFFUSIVE CONVECTION" J FLUID MECHANICS, (1976) 78, 821-854.

- LINEAR STABILITY ANALYSIS



DERIVATION OF EQUATIONS

BACKGROUND STATE

PERTURBATION EQUATIONS

BOUSSINESQ APPROXIMATION

DIMENSIONLESS VARIABLES

BOUNDARY CONDITIONS

$$\left(\frac{1}{P_r}\frac{\partial}{\partial t} - \nabla^2\right)\nabla^2\psi = -R_a\frac{\partial T_1}{\partial x} + R_s\frac{\partial S_1}{\partial t}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)T_1 = -\frac{\partial \psi}{\partial x}$$

$$\left(\frac{\partial}{\partial T} - \nabla^2\right)S_1 = -\frac{\partial \psi}{\partial x}$$

SOLUTION

$$\psi(x,z,t) = A e^{pt} \sin(\pi a x) \sin(\pi n z)$$

$$T_1(x,z,t) = B e^{pt} \cos(\pi a x) \sin(\pi n z)$$

$$S_1(x,z,t) = C e^{pt} \cos(\pi a x) \sin(\pi n z)$$

DISPERSION RELATION

$$p^3 + (1 + \tau + P_r) k^2 p^2$$

$$+ \left[(\tau + P_r \tau + P_r) k^4 - (R_a - R_s) \frac{\pi^2 a^2}{k^2} P_r \right] p$$

$$+ \tau P_r k^6 + \pi^2 a^2 P_r (R_s - \tau R_a) = 0$$

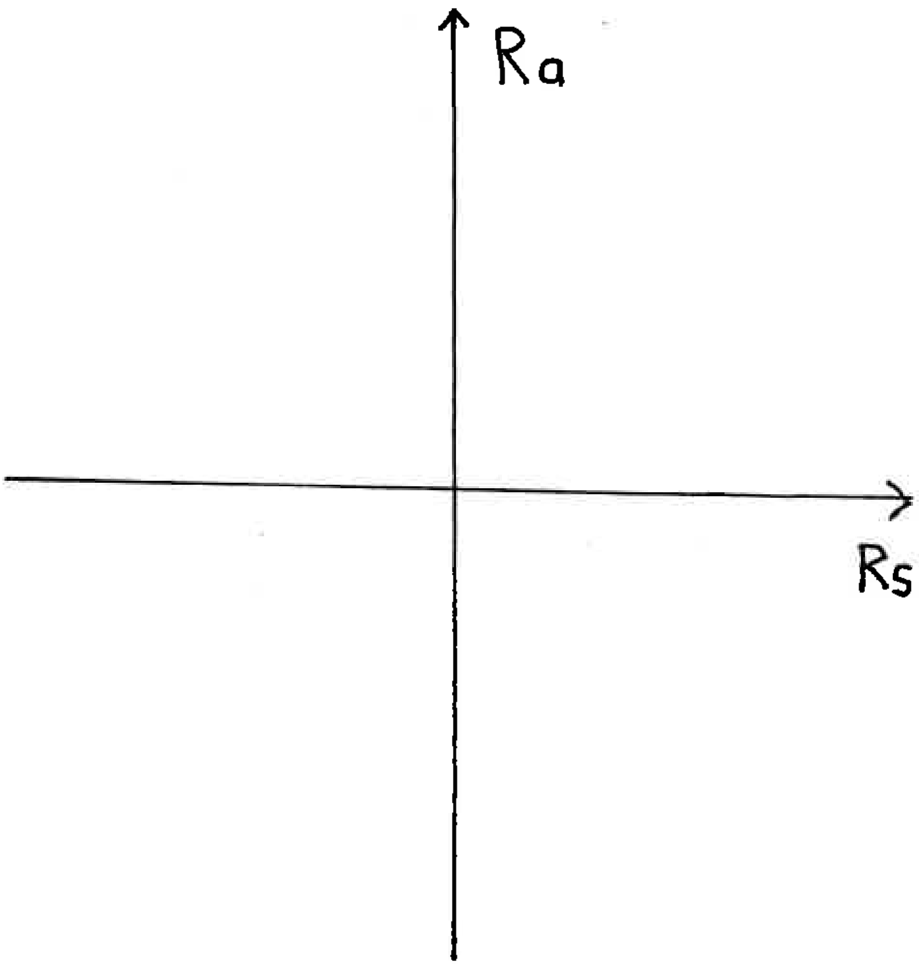
$$k^2 = \pi^2 (a^2 + n^2)$$

$$p = p_R + i p_I$$

MARGINAL STATE $p = 0$ SALT FINGERS

$p = i p_I$ OSCILLATORY MOTION

INVESTIGATE PROPERTIES OF THE DISPERSION RELATION IN THE (R_s, R_a) PLANE



- REGIONS OF STABILITY
- REGIONS OF INSTABILITY
- REGIONS OF OSCILLATORY MOTION

APPLICATIONS

LAKE KIVU

- ON BORDER BETWEEN RWANDA AND CONGO
- METHANE AND CARBON DIOXIDE DISSOLVED IN LAKE
- METHANE EXTRACTED TO GENERATE ELECTRICITY
- UNDERSTANDING THE STABILITY AND DEGASSING OF THE LAKE IS VERY IMPORTANT

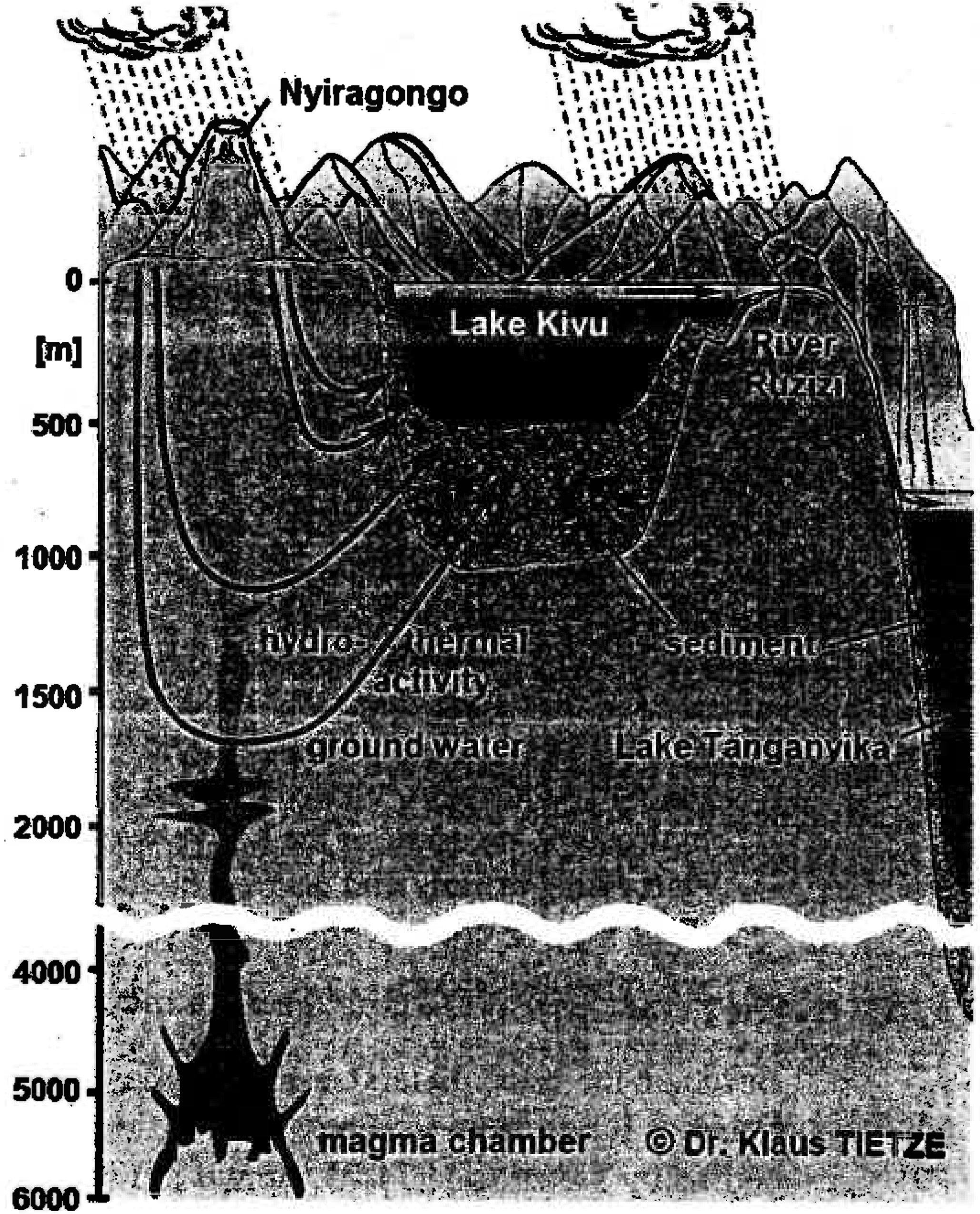
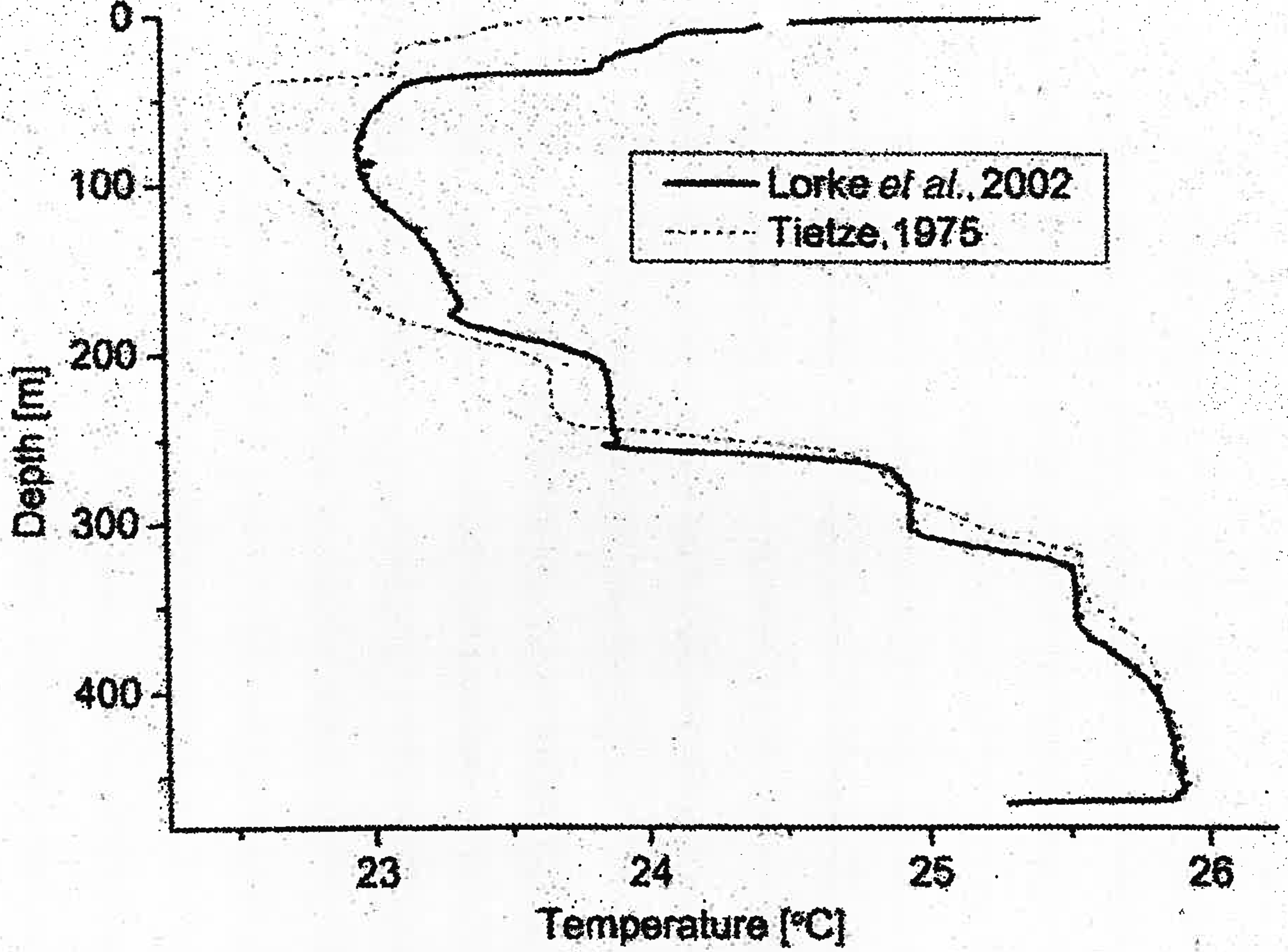
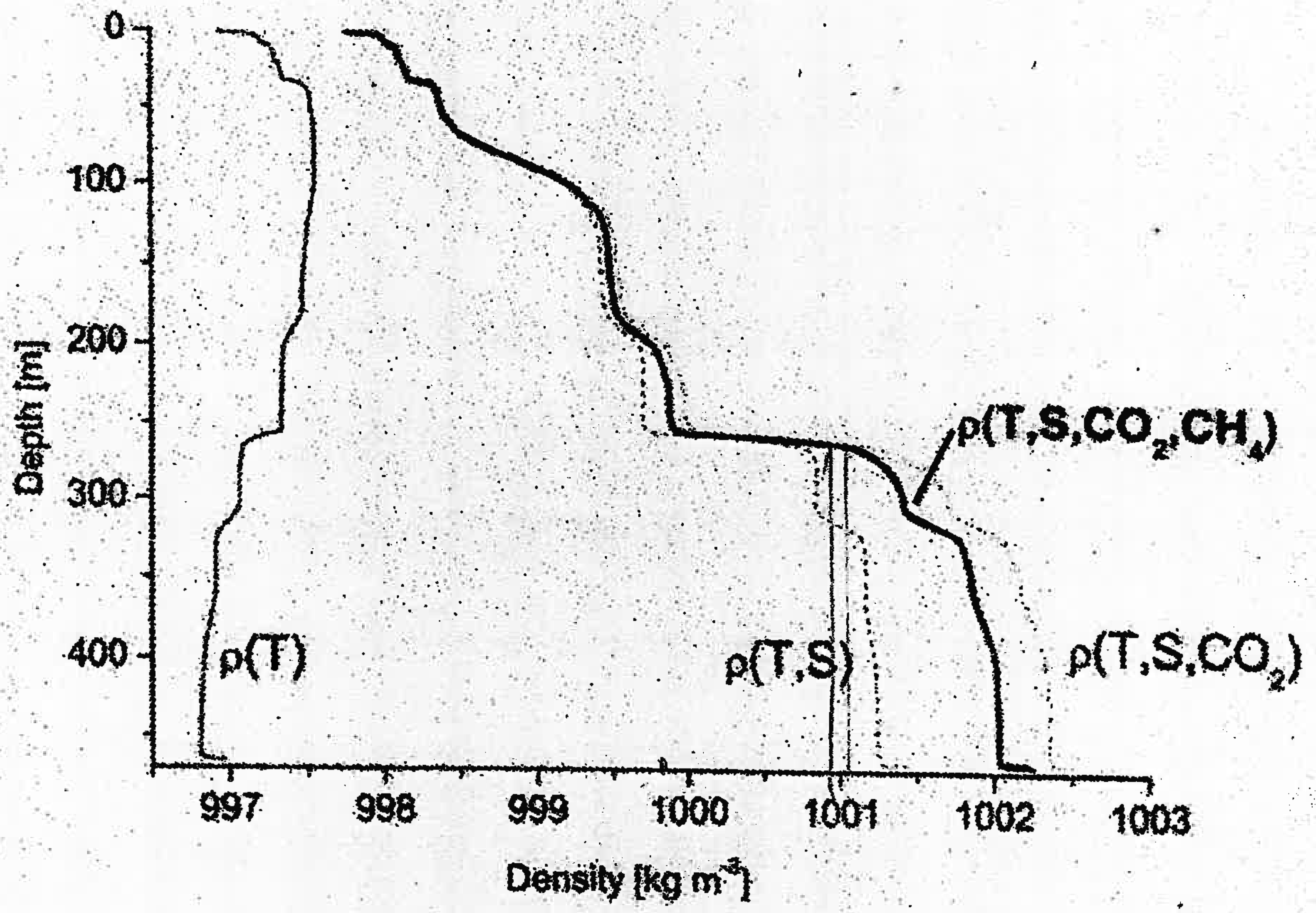


Figure 4 *Schematic cross section through Lake Kivu and Lake Tanganyika showing the mode of influx and infiltration of fresh and saline water into Lake Kivu (after Tietze, 2005).*


Lorke et al., 2002
Tietze, 1975
Depth [m]
0
100
200
300
400
23
24
25
26
Temperature [°C]


Depth [m]
0
100
200
300
400
997
998
999
1000
1001
1002
1003
Density [kg m^{-3}]
ρ(T)
ρ(T,S)
ρ(T,S,CO_2,CH_4)
ρ(T,S,CO_2)

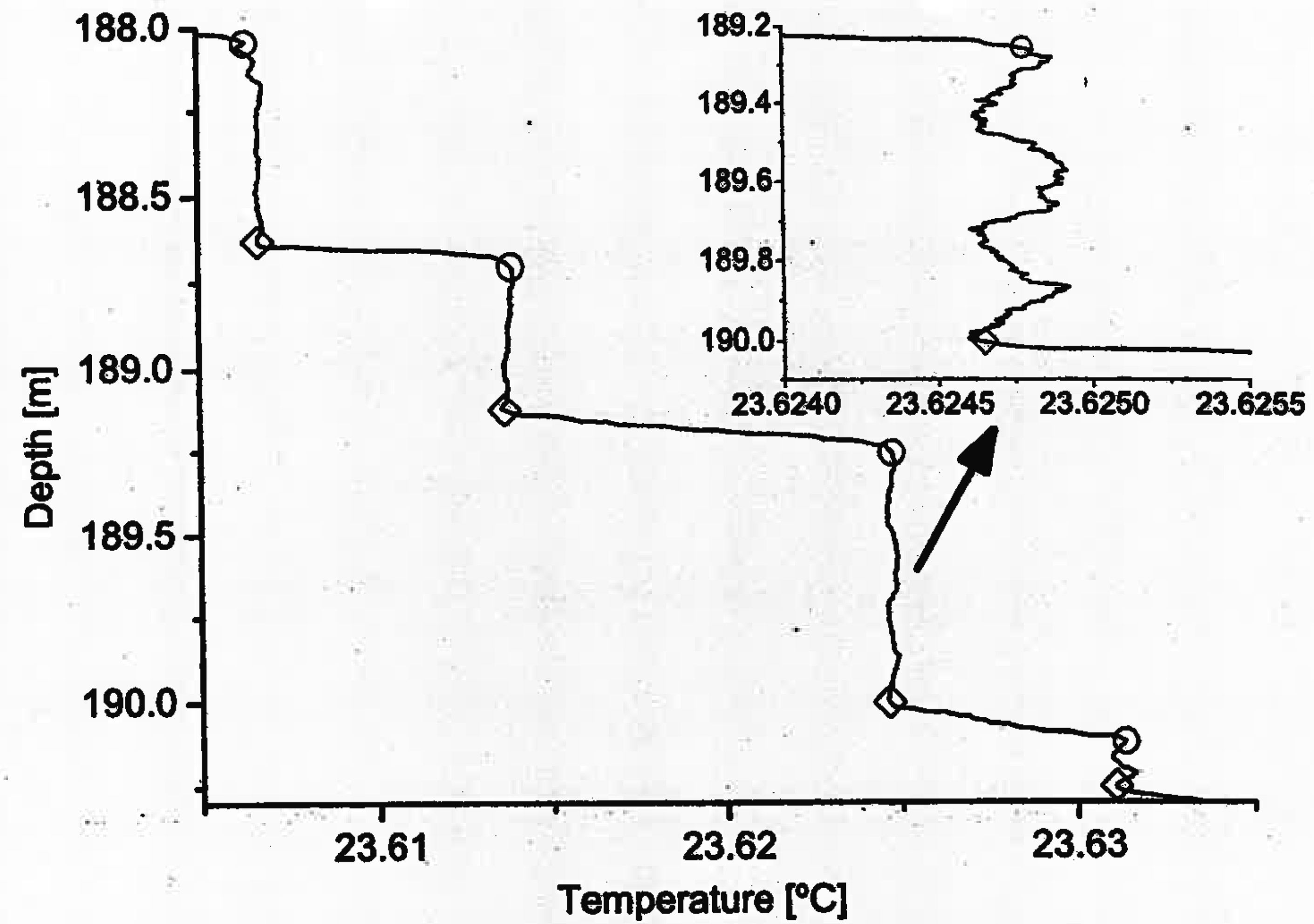


Figure 3. An example of double-diffusive layering observed in Lake Kivu at 188 to 190 m depth. The microstructure profiles were split into well-mixed layers and interfaces as depicted by the open circles (upper boundaries of mixed layers) and diamonds (lower boundaries of mixed layers). The sharpness of the transition between layers evidences the current activity of the double-diffusion. The inset shows the observed temperature variability within one of the mixed layers.

OTHER APPLICATIONS

- LAYERS OF LAVA IN AN IGNEOUS INTRUSION

Figure 9. Layers in the Upper Zone of the Skaergaard igneous intrusion, made visible by contrasting light and dark colours of different minerals (from McBirney & Noyes 1979).

• TEMPERATURE STEPS UNDER AN ARCTIC ICE ISLAND

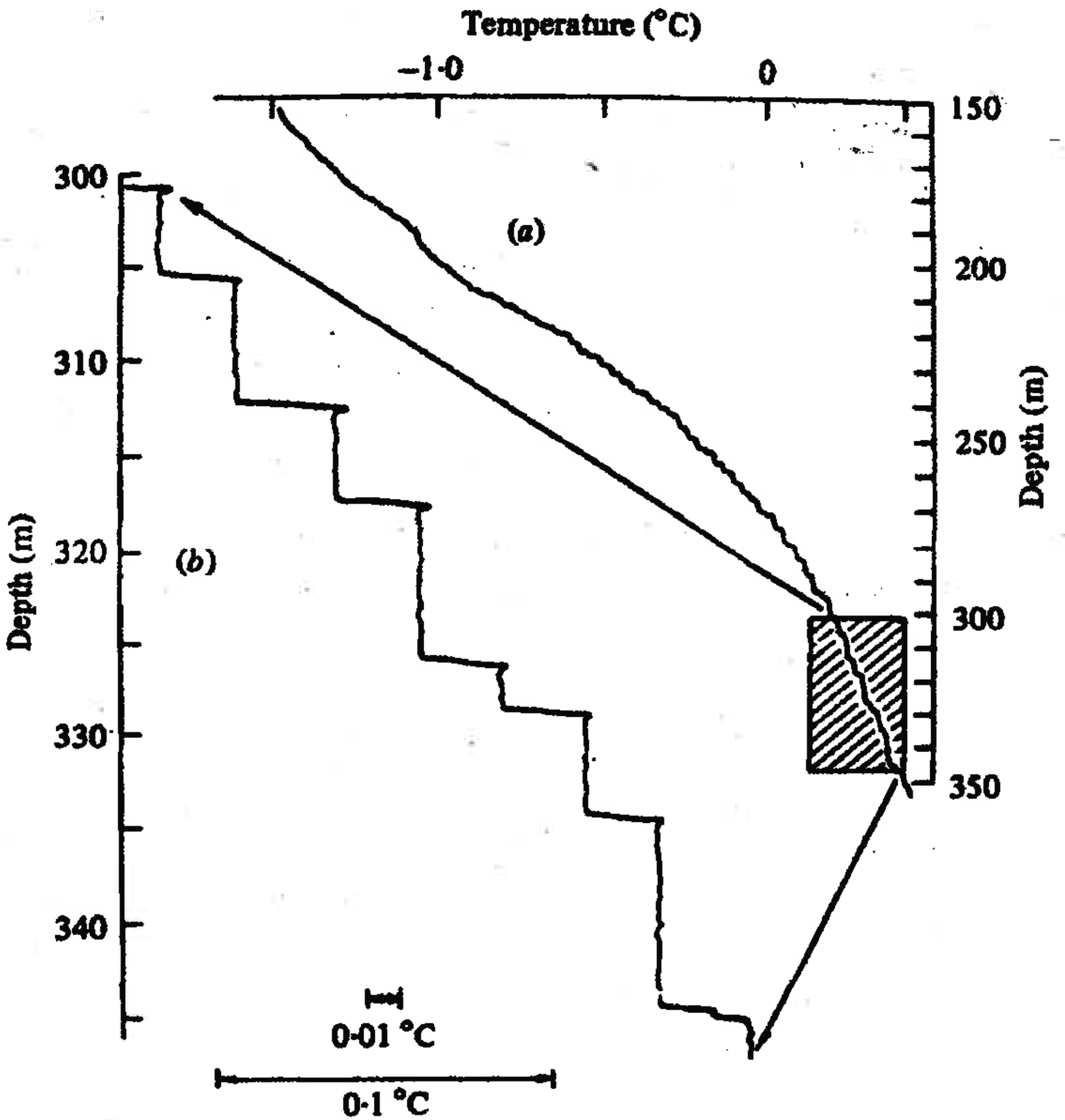


FIGURE 8. A temperature profile obtained under an Arctic ice island by Neal, Neshyba & Denner (1969) showing steps formed by the double-diffusive mechanism. (*a*) Typical temperature profile section. (*b*) Section of profile recorded at high gain.

- SALT FINGERS

  POURING SALT SOLUTION ON TOP

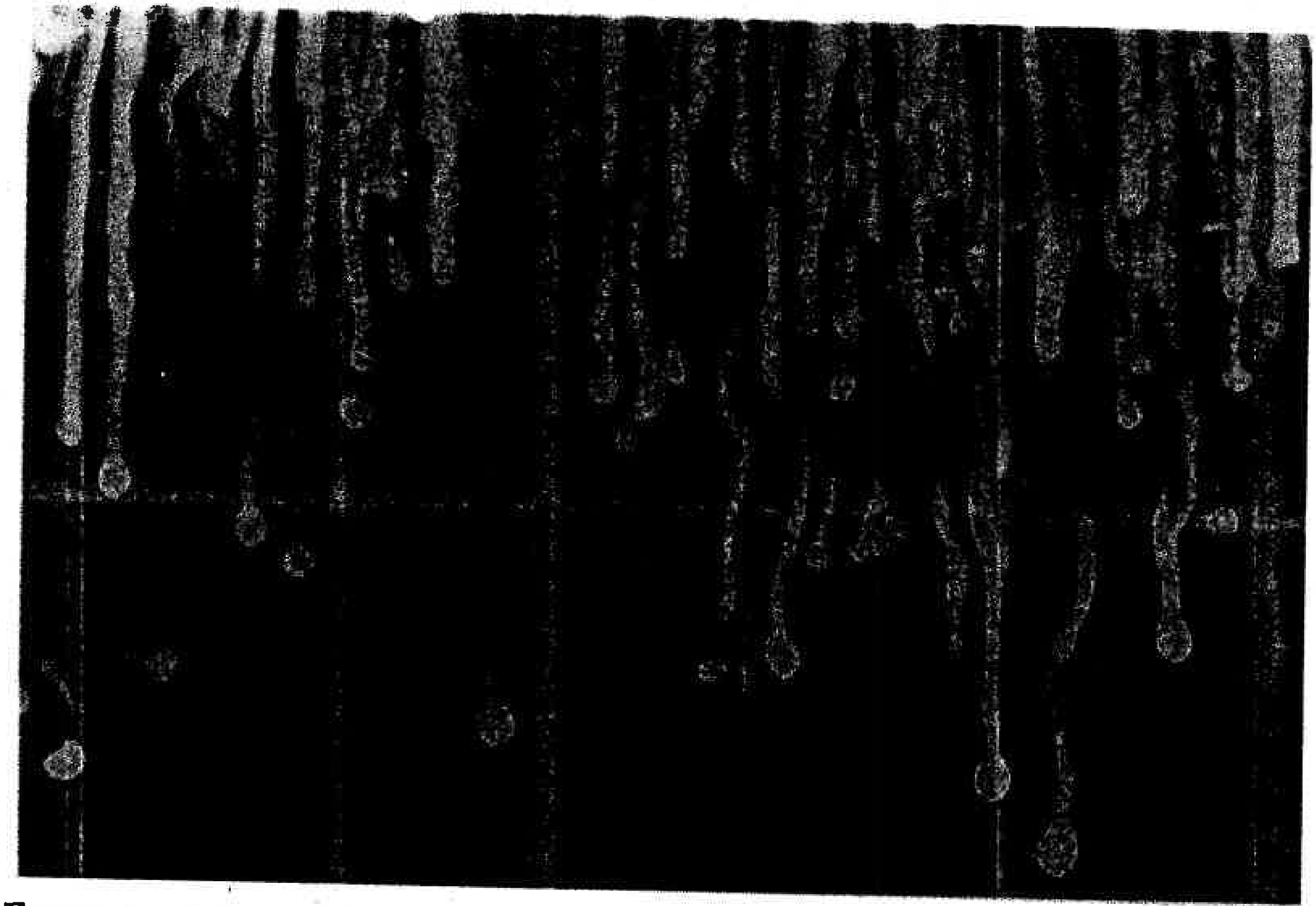


FIGURE 1. A field of salt fingers formed by setting up a stable temperature gradient and pouring a little salt solution on top. The downward-moving fingers were made visible by adding fluorescein to the salt and lighting through a slit from below.

- SALT FINGERS

  UPWARD MOVING FINGERS

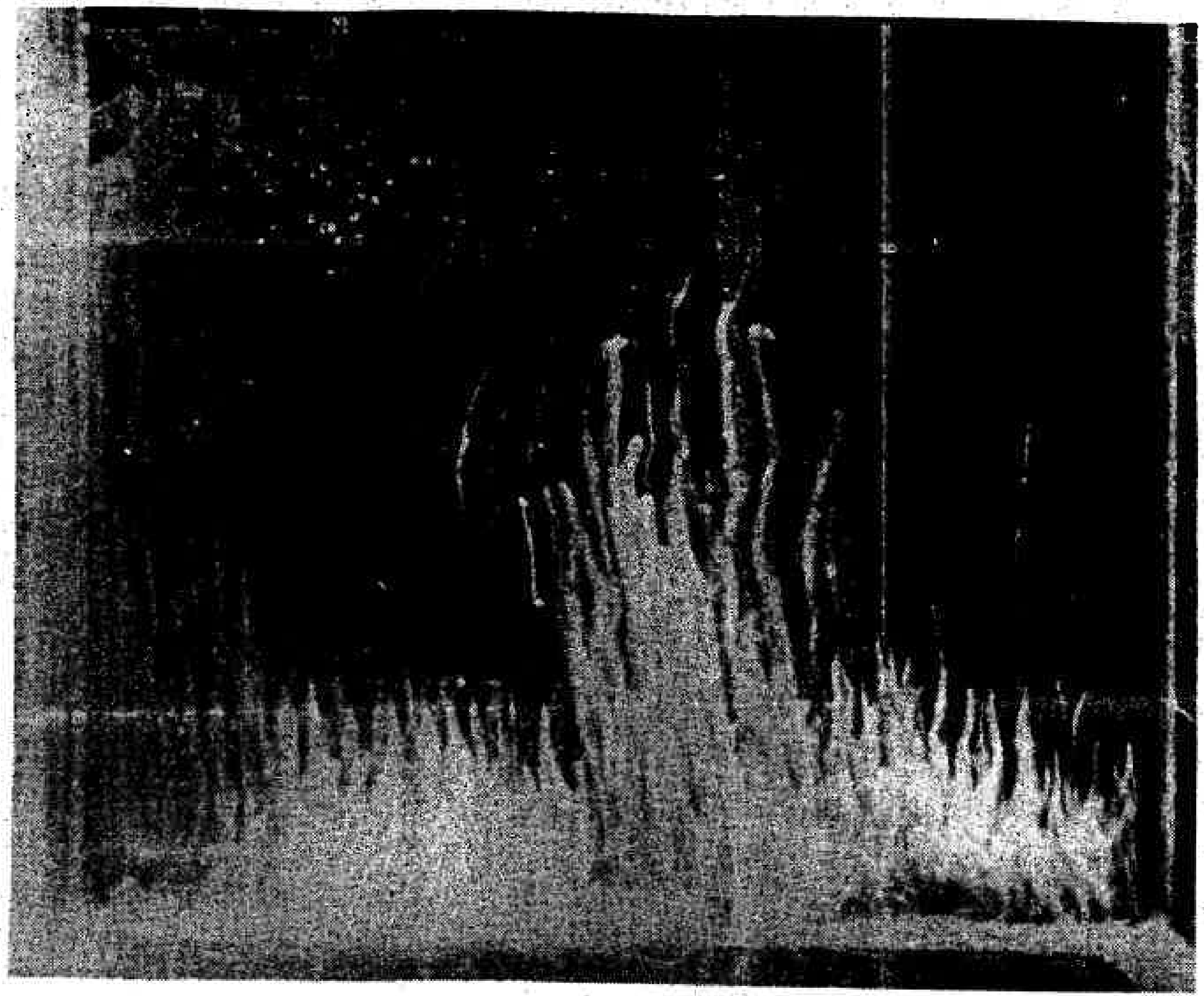


Figure 3 Vertical cross section of salt fingers, marked by fluorescein dye added to the upward moving fingers. (The experimental tank is 25 cm wide.)